



Techniques of life table construction: A review

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Abstract

A life table is one of the most influential discoveries in demography which is primarily used to express the pattern of human mortality. Among the various uses, it plays an imperative role for analysis of mortality, population projection, life expectancy design and to determine the key factors responsible for the highest mortality within the population. Public health employees, demographers, actuaries and many others use life tables for innumerable purposes. A life table can be constructed for a country or an area on the basis of various demographic characteristics including sex, occupation, race and so on. It can be classified according to reference year of the table, age detail and a number of factors comprehended by the table. Therefore, the purpose of this paper is to introduce life table including its various types and construction procedures in the understanding ways.

Keywords: life table; mortality schedule; abridged life table; complete life table; life expectancy

Introduction

In demography, a life table provides an estimate of the probability that a member of the cohort will die at a precise age. It affords to understand the changes in population due to different ages throughout their lives which are associated with the mortality rates at different ages. It is an analytical technique in studying the calculation of mortality of an organism and individuals (Siegel & Swanson, 2004) ^[15]. It is also concerned with survival and life span, so it is viewed as 'mortality table' (Etikan, Abubakar & Alkassim, 2017) ^[11]. The assumptions to which life table is based on may not be attainable in actual life but still, it has proved itself to be the most beneficial and authentic model and no superior model has been recommended to study various demographic components.

At first Graunt (1662) ^[2] has introduced the life table concept and tabulated the number of deaths based on the analysis of 'Bills of Mortality'. That life table was found defective since it has been constructed based on mortality experience only (Ramakumar, 1986) ^[11]. Halley (1693) ^[4] first informed how to construct and calculate a life table properly and published the first recognized life table based on birth and death registration data for Breslau. The life table was contained most of the columns of modern life table and the life table we use today methodologically is similar to Halley's life table but his life table was not correct as it was constructed based on the assumption that population remained stationary (Siegel & Swanson, 2004) ^[15]. In the 17th and 18th centuries, several attempts were made to construct a life table based on limited data. At first Milne (1815) ^[7] published a life table using population and death data by age. King (1912) was the first person to show us a method of getting the rates of the life table at pivotal values using interpolation formula. A short method for calculation of abridged life table was given by Reed and Merrell (1939) ^[12] and Chiang (1960) suggested a method to calculate person-years lived. During the past 100 years, there were many contributions to the development of life table by demographers, actuaries, and mathematician and so on.

Types of life table

According to reference year of the table, life table can be distinguished into two types: the generation or cohort life table and the current or period life table. Again, it can be classified into two categories: complete or unabridged and abridged life table according to the length of the age interval in which the data are presented. Moreover, single or multiple decrement life tables are also considered according to the number of decrements characteristics considered in a life table.

Cohort life table

Cohort life table reflects the summary presentation of the death history of the actual cohort. According to this type of life table, the death history of the actual cohort would be observed from their moment of birth through each successive age and continue until all of them die. It requires a long period of time as the span of the cohort can be anywhere near 100 years or more. However, cohort life table is suitable to study mortality of those who possess short span of life like insects and plants. It should be kept in mind that cohort life

expectancies tend to be higher than period life expectancies because they include any assumptions about future improvements in mortality rates.

Period life table

Period life table represents the mortality experience of the synthetic cohort over a short period of time such as 1 year, 3 years or an intercensal period if it experienced the mortality conditions of a particular period. This is also referred to as cross-section life table or current life table (Rowland, 2003) [13]. A period life table is constructed using current population data. This type of life table signifies mortality experience by age of the members of the hypothetical cohort rather than actual cohort in a particular period of time. Therefore, period life table may be viewed as an excellent summary description of mortality for a short period of time.

Complete life table

Complete life table presents decrementation due to deaths and other functions of life table for a single year of age from birth to the last applicable age. This type of life table can be constructed from the observed data or by suitable interpolation of an abridged life table (Swanson & Siegel, 2004) [15].

Abridged life table

A life table in which value of the life table functions is presented for a certain age group usually 5 or 10 years intervals are termed as abridged life table. An abridged life table is found to be satisfactory and convenient for most of the situations in demographic analysis.

Single decrement life table

Single decrement life table concerns with single attrition factor and considers only one characteristic at a time. This type of table is a process in which the members of the cohort have only one recognized mode of departure from a well-defined state. Mortality is one such process.

Multiple decrement life table

Multiple decrement life tables summarize the mortality experience of cohort where the members of the cohort lost their membership cause of two or more attrition factors (Hinde, 1998) [5]. In such table, separate and combined effects of more than one factor are treated.

Life table construction methods

A life table is constructed based on age-specific death rates in a specific population. As it is constructed to trace the life history of the cohort, it is assumed that all those born should die and no one can be included at an age in between. To construct both single and multiple decrement life tables, there are required some assumptions; the cohort is originated from some standard number of births say 10,000 or 1,00,000 which is called the radix of the life table. As mortality experience is found to be different for males and females, a life table is constructed separately for them. Death is considered uniformly distributed within the interval and the members of the cohort will die according to pre-determined mortality schedule at each age. A single decrement life table includes the following functions:

[1]	[2]	[3]	[4]s	[5]	[6]	[7]	[8]
$(x, x+n)$	${}_nq_x$	${}_np_x$	l_x	${}_nd_x$	${}_nL_x$	T_x	e_x^0

The functions shown in the table are briefly described below:

Column 1, Age interval, $(x, x+n)$: Age interval from exact age x to age $x+n$, where n is a positive integer.

Column 2, Probability of dying (${}_nq_x$): The probability that the persons in the cohort alive at the beginning of age interval x and would die during the next age.

The value of ${}_nq_x$ is obtained using the following formula:
$${}_nq_x = \frac{{}_nm_x}{1 + (n - a_x){}_nm_x}$$

where n refers the length of age interval, ${}_na_x$ refers the person-years lived in the age interval x to $x+n$ by those dying in the interval (Preston *et al.*, 2001), and ${}_nm_x$ is the observed mortality rate.

The other formulas are used for transforming the value of observed mortality rate to the probability of dying are given below:

$$q_x = \frac{2m_x}{2+m_x} = \frac{d_x}{l_x} = \frac{d_x}{L_x + \frac{1}{2}d_x}$$
 For complete life table, where ${}_nd_x$ refers the number of deaths between exact ages x and $x+1$, m_x refers the

observed death rates between exact ages x and $x+1$, l_x refers the total number of person survive at exact age x , and L_x refers the number of person-years lived between ages x and $x+1$. Again,
$${}_nq_x = \frac{2n{}_nm_x}{2+n{}_nm_x} = \frac{{}_nd_x}{l_x}$$
 for abridged life table, where ${}_nd_x$ refers the number of deaths between exact ages x and $x+n$.

Column 3, Probability of surviving (${}_n p_x$): The probability of surviving from exact age x to $x + n$ and it can be written as follows:

$p_x = 1 - q_x = \frac{l_{x+1}}{l_x}$, for complete life table, where l_{x+1} refers the total number of person survive at exact age $x + 1$. Again,

${}_n p_x = 1 - {}_n q_x = \frac{l_{x+n}}{l_x}$, for abridged life table, where l_{x+n} refers the total number of person survive at exact age $x + n$. Also, for open age

interval: ${}_{\infty} p_x = 0$.

Column 4, Survivors (l_x): The number of persons living at the beginning of age interval out of assumed number of birth l_0 , which is the radix of the life table which is assumed number of birth to age 0 and is usually taken as 1,00,000. The l_x is also known as decreasing function and can be calculated as following formula:

$l_{x+1} = l_x \times p_x = l_x(1 - q_x) = l_x - d_x$, for complete life table, and $l_{x+n} = l_x \times {}_n p_x = l_x(1 - {}_n q_x) = l_x - {}_n d_x$, for abridged life table.

Column 5, Number of deaths (${}_n d_x$): The number of deaths between exact ages x and $x + n$ out of total number of births assumed in the table. The ${}_n d_x$ value can be calculated as, $d_x = l_x - l_{x+1} = l_x q_x$, for complete life table, and ${}_n d_x = l_x - l_{x+n} = l_x \cdot {}_n q_x$, for abridged life table.

Column 6, Person-years lived (${}_n L_x$): Number of person-years lived by the cohort assumed number of birth l_0 between ages x and $x + n$. it is also called the life table population (Pollard *et al.*, 1974)^[8]. Mathematically it can be written as

${}_n L_x = n \cdot l_{x+1} + {}_n a_x \cdot d_x$ when l_x is supposed linear, L_x can be obtained as $L_x = \frac{l_x + l_{x+1}}{2}$.

But linear relationship is invalid for age 0 and 1 and in such case, the following formula is suggested for obtaining the value as $L_0 = 0.3l_0 + 0.7l_1$ and $L_1 = 0.4l_1 + 0.6l_2$ where l_0 is radix.

For abridged life tables, the value of ${}_n L_x$ can be obtained by following formula:

$${}_x L_n = \frac{n(l_x + l_{x+n})}{2}$$

$$L_0 = 0.3l_0 + 0.7l_1$$

$${}_4 L_1 = \frac{4}{2}(l_1 + l_5)$$

$${}_{\infty} L_x = \frac{l_x}{{}_{\infty} m_x}, \text{ for open age interval.}$$

Here ${}_{\infty} L_x$ refers the number of person-years lived at open age interval and ${}_{\infty} m_x$ refers the observed death rate at open age interval.

Column 7, Person-years lived after age (T_x): This column represents the total number of person-years that would be lived by the cohort l_x after attaining the age x . mathematically, $T_x = \sum_{a=0}^{\infty} L_{x+a}$, For complete life table, and $T_x = \sum_{a=x}^{\infty} {}_n L_a$, for abridged life table.

Column 8, Complete expectation of life (e_x^0): The last column of life table implies the average remaining life time in years for a person aged x can expect to live, including part years. It is defined mathematically as the following formula: $e_x^0 = \frac{T_x}{l_x}$.

It is related to the curtate expectation of life by the following formula: $e_x^0 = e_x + \frac{1}{2}$.

Here e_x is the curtate expectation of life which represents the average number of complete years of life lived after attaining age x . To

calculate the value of curtate expectation of life, the following equation is suggested, $e_x = \frac{\sum_{t=1}^{\infty} l_{x+t}}{l_x}$. It is also related to the probability of surviving by the following formula: $e_x = p_x(1 + e_{x+1})$.

Short-cut Methods

Some sophisticated short-cut methods used for the construction of abridged life table are: i. Reed-Merrell method, ii. Greville’s method, iii. Keyfitz-Frauenthal method, and iv. Methods of reference to a standard table.

Reed-Merrell method

Reed and Merrell (1939)^[12] introduced a shortcut method for calculating abridged life table to give more accurate results. In this method, at first it is required to convert the value of observed mortality rate (${}_n m_x$) to the probability of dying (${}_n q_x$) and the rest of the functions

of life table are calculated on the basis of ${}_n q_x$ values. The value of ${}_n q_x$ is obtained by the following exponential equation:

$${}_n q_x = 1 - e^{-n \cdot m_x - a n^3 m_x^2};$$

where n is the size of the age interval, ${}_n m_x$ is the observed death rate and a constant a value of a=.008

gives acceptable results. After calculation of ${}_n q_x$, the value of l_x and ${}_n d_x$ are calculated by using the following formulas: $l_{x+n} = l_x \times {}_n p_x$

$$\text{and } {}_n d_x = l_x - l_{x+n}.$$

Further, T_x values are directly obtained from l_x values for 5 or 10 years age interval by using the following equation:

$$T_x = -0.20833l_{x-5} + 2.5l_x + 0.20833l_{x+5} + 5 \sum_{i=1}^{\alpha} l_{x+5i};$$

for 5-year age interval and

$$T_x = 4.166667l_x + 0.83333l_{x+10} + 10 \sum_{i=1}^{\alpha} l_{x+10i};$$

for 10-year age interval.

To obtain the value of ${}_n L_x$ under age 10, Reed and Merrell (1939)^[12] suggested the following equations:

$$L_0 = 0.276l_0 + 0.724l_1,$$

$$L_1 = 0.410l_1 + 0.590l_2,$$

$${}_4 L_1 = 0.034l_0 + 1.184l_1 + 2.782l_5,$$

$${}_5 L_5 = -0.003l_0 + 2.242l_5 + 2.761l_{10}.$$

However, for ages 10 years and over ${}_n L_x$ values are obtained from the differences between the T_x values and e_0^x is calculated as the ratio of T_x and l_x .

Greville’s method

Greville (1943)^[3] suggested a method to construct an abridged life table in an easy way. In this method, the following equation is used to convert the value of observed central death rate (${}_n m_x$) to the probability of dying (${}_n q_x$):

$${}_n q_x = \frac{{}_n m_x}{\frac{1}{n} + {}_n m_x \left[\frac{1}{2} + \frac{n}{12} ({}_n m_x - \ln C) \right]}$$

where $\ln C$ is assumed to be about 0.095. In the Greville’s method, it is recommended that the value of ${}_n m_x$ both in life table and

observed population are same (Greville, 1943)^[3]. Further, ${}_n L_x$ column is obtained using the following equation: ${}_n L_x = \frac{{}_n d_x}{{}_n m_x}$,

$${}_{\infty} L_x = \frac{l_x}{{}_{\infty} m_x};$$

for open age interval. The rest of the functions are calculated as usual notation of life table.

Keyfitz-Frauenthal method

Keyfitz and Frauenthal (1975)^[6] method is considered as the simplification of the Greville method (Poston & Micklin, 2006)^[9]. Keyfitz and Frauenthal (1975)^[6] suggested the following procedure to convert the value of observed death rate (${}_n m_x$) to the probability of dying

$$({}_n q_x) \text{ on which the accuracy of life table depends: } {}_n q_x = 1 - \exp \left[-n \left({}_n m_x + \frac{({}_n p_{x-n} - {}_n p_{x+n})({}_n m_{x+n} - {}_n m_{x-n})}{48 {}_n p_x} \right) \right];$$

where p is observed population.

The desired value of ${}_n L_x$ is calculated using the following formula: ${}_n L_x = \frac{n(l_x - l_{x+n})}{\ln l_x - \ln l_{x+n}} \left[1 + \frac{n}{24} ({}_n m_{x+n} - {}_n m_{x-n}) \right].$

Further, T_x and e_x^0 are calculated using as usual way.

Method of reference to a standard table

Abridged life tables can be constructed by reference to a standard table. According to this method, mortality is assumed to be at a comparable level in both tables. Now the value of ${}_n g_x$ is obtained using ${}_n q_x$ and ${}_n m_x$ of the standard life table by following formula:

${}_n g_x = \frac{n}{{}_n q_x} - \frac{1}{{}_n m_x}$; where ${}_n g_x$ signifies the average number of years lived in the interval of age group by those dying in the interval

(Sirken, 1966)^[14]. The ${}_n q_x$ values for new life table are obtained by using the following formula: ${}_n q_x = \frac{{}_n m_x}{1 + {}_n g_x \cdot {}_n m_x}$.

For the calculation of ${}_n L_x$, first it is required to compute a factor may be designated by ${}_n G_x$ using the following formula:

${}_n G_x = \frac{{}_n l_x - {}_n L_x}{{}_n d_x}$; where ${}_n G_x$ signifies the distribution of deaths in the interval x to $x + n$ (Sirken, 1966)^[14]. Now ${}_n L_x$ values for new

life table is obtained as ${}_n L_x = {}_n l_x - {}_n G_x \cdot {}_n d_x$. for open-ended age group, ${}_\infty L_x$ is computed from a special formula. In that case, a factor

r_x is required to calculate for standard life table by using the following formula: $r_x = \frac{{}_\infty m_x \cdot {}_\infty L_x}{l_x}$. Now ${}_\infty L_x$ for new life table is

calculated as follows: ${}_\infty L_x = \frac{l_x \cdot r_x}{{}_\infty m_x}$. Other functions of life table can be calculated as usual ways.

Furthermore, some life tables recognized as model life tables have been indicated in this study are: i. United Nations model life tables, ii. Coale and Demeny regional model life tables, iii. Ledermann's system of model life tables, iv. Brass logit-life table system, and v. United Nations model life tables for developing countries.

Conclusions

A life table is a design that displays various pieces of information about mortality experience of cohort and can be functional in an extensive variety of population studies. In this study, various understanding methods have been discussed for constructing both complete and abridged life tables. Moreover, four shortcut methods used for transformation from observed mortality rate (${}_n m_x$) to probability of

dying (${}_n q_x$) for constructing abridged life tables are also briefly narrated. Using this method, life tables can be constructed for both

single and group data for various purposes of a country.

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